A Node Localization Algorithm based on Wireless Sensor Network

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Abstract

After analyzing the disadvantages of the centralized multidimensional localization algorithms MDS-MAP (Multi-Dimensional Scaling-MAP) in positioning accuracy and computational complexity, we present a new localization algorithm based on a set of statistical vectors (SV). The solving equation of the double center matrix can be simplified by node coordinate transformation. In order to reduce the noise disturbance and decrease the effect of ranging error on the followed location accuracy, a new coordinate inner product matrix can be reconstructed by using a set of statistical vectors, which can be used to calculate the node coordinates directly. This algorithm can realize centralized localization, distributed localization and incremental localization of nodes.

Keywords: wireless sensor network; node; localization algorithm; SV positioning algorithm

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1. Introduction

During the WSNs positioning, generally it assumes neighboring nodes can communicate with one another and the distance among them can be surveyed. Basically, the smaller the measured distance error, the higher the positioning precision can achieve [11,14]. So, it’s hoped that the distance measurement error can be decreased by modifying node’s hardware structure or increasing node’s transmitting power [2,7]. However, high precision distance measuring means will bring about increased hardware cost and energy consumption. For instance, the distance measurement technique TOA requires strict time synchronization. TDOA requires two signal transmitting devices; AOA needs multiple signal receiving devices. Increasing node’s transmitting power will shorten the shelf life of wireless sensor nodes. All those go against the design principle of low cost and low consumption of WSNs [5,12,13].

Among the four distances ranging techniques like RSSI, TOA, TDOA and AOA, the positioning system based on RSSI is at the lowest hardware cost and can be easily implemented [10]. That’s why it’s favored by scholars home and abroad. But comparatively, it causes the biggest distance measuring error. RSSI’s basic principle is to convert the attenuation of signal strength to signal propagation distance; with extending of transmitting distance, the signal intensity of wireless signal would be decay according to certain rules. Yet, the propagation feature of signal is related with environmental parameters; signal damping is affected by many factors such as temperature, humidity, obstacles, propagation mode and multipath effect [3,8]. Hence, it’s quite difficult to establish a uniform signal transmit attenuation model. Besides, the height of radio frequency antenna is another important factor affecting signal transmits attenuation model. The precision of positioning algorithm based on RSSI is associated not only with the distance measuring accuracy and also with features of the positioning algorithm itself. In academic circles, lots of positioning algorithms based on RSSI were proposed, e.g. well-known RADAR positioning system, multiple point distance measurement positioning method, maximum posterior probability position technique based on model and map [6].

The paper is designed to discuss positioning problem based on RSSI distance measurement. In order to reduce the influence of distance measuring error on positioning precision, it proposed the positioning algorithm based on statistical

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vector set (SV). Through coordinate conversion, the method simplifies the solution equation for centralized coordinate matrix and utilizes statistically uncorrelated vector set to re-construct coordinate inner product matrix, reducing the noise disturbance in range measurement. With direct use of coordinate inner product matrix, the coordinate of node can be calculated. Experiment shows that in the case of big distance measuring error, SV positioning algorithm can effectively improve positioning precision, rather suitable for node positioning based on WSNs with lower cost hardware.

Suppose communication distance among all nodes is the same in the network. Nodes between two can communicate through one or more hops, and each node can send and receive signals and has function of calculating signal strength. In the paper, CC2430 of Texas Instrument is used as main chip of sensor node as to receive signal. In order to validate the practicability of the proposed algorithm, considering the error of RSSI signal would be bigger, here in the simulation part, we use Noisy Disks model to emulate any possible distance measuring error caused by RSSI in the way of directly increasing distance measuring error.

2. SV positioning algorithm

2.1. Centralized SV positioning algorithm

Centralized positioning algorithm is the kind of technique which concentrates all required information to one central node of computer then performs positioning of single or all nodes from there. Suppose original coordinate of N nodes \{X_1, X_2, ..., X_N\} is \(X_i \in \mathbb{R}^d\) \(\forall i \in \{1, 2, ..., N\}\) means the collection of sensor nodes. Translate all nodes to make \(X_i\) at original point of coordinate and the translation amount is \(a = (a_1, a_2, ..., a_d)\); then define new coordinate as \(x_i (x_i \in \mathbb{R}^d)\), where \(X_i\) can be any node of referential nodes; then after translation, the coordinate matrix of referential node is \(X_N = [X_1, X_2, ..., X_N]^T\). The two-two distances among all nodes constitute measuring matrix \(D_N = (\tilde{d}_{ij})_{M \times N}\), then measuring distance among nodes is shown in Equation (1).

\[
\tilde{d}_{ij} = N(d(x_i, x_j), \sigma)
\]

\(d(x_i, x_j)\) is the Euclidean distance between \(X_i\) and \(X_j\) for the sensor nodes. In order to get the coordinates of the nodes between the inner product, the introduction of coordinate inner product matrix \(B_N = (b_{ij})_{N \times N}\), \(b_{ij}\) is shown in Equation (2).

\[
b_{ij} = (\tilde{d}_{ij}^2 + \tilde{d}_{ji}^2 - \tilde{d}_{ij}^2) / 2
\]

Since measured value \(\tilde{d}_{ij}\) has error, the correlation of data which are interrelated at \(d\) dimension is forced to extend towards high dimension and decompose characteristic value of \(B_N\), the number of characteristic value which is bigger than 0 will be more than \(d\); if the first \(d\) feature values and relevant feature vector are chosen to form according coordinates, the error will be bigger.

In WSNs, since \(B_N\) is usually singular, there is at least one characteristic value to 0 as to do characteristic decomposition of \(B_N\), which will cause nonexistence of inverse matrix of feature value matrix. The characteristic decomposition of \(B_N\) is expressed in Equation (3).

\[
B_N = (V_1, V_2) \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}
\]

The coordinates of unknown nodes can be calculated by \(X_N X_N^T = b_k\). It is shown in Equation (4).

\[
x_i = (X_N X_N)^+ X_k b_k
\]
To sum up the derivation process, the centralized SV location algorithm includes the following steps:

Step 1: Translation of the reference node so that a reference node is located in the origin of the coordinates.

Step 2: Calculate the distance two points according to the signal strength nodes.

Step 3: Calculate the coordinates of the inner product matrix using Equation (3).

Step 4: To coordinate the inner product matrix center matrix decomposition, take more than zero eigenvalues of the coordinates of the inner product matrix feature vector is constructed and the corresponding.

Step 5: Calculate the unknown node coordinates using Equation (4), and translate them into the original coordinate system.

2.2. Distributed SV positioning algorithm

The distributed positioning algorithm needs to communicate only with neighboring nodes during positioning. The positioning process is completed within the nodes. In the distributed SV positioning algorithm, it uses only the coordinate of anchor nodes and the distance information between specified unknown nodes and anchor nodes to realize distributed positioning of the specified unknown nodes.

Suppose in WSNs, there are K anchor nodes like \( X_1, X_2, \ldots, X_K \) and original coordinate is \( d \in \mathbb{R}^d \); \( \{1, 2, \ldots, K\} \) means the collection of K anchor nodes. Do coordinate translation of all anchor nodes to make \( X_1 \) at original point of anchor and translation amount is \( a = (a_1, a_2, \ldots, a_K) \); new coordinate is defined as \( x' \in \mathbb{R}^d \), where \( X_1 \) can be any of anchor nodes; then after translation, the coordinate matrix of anchor nodes is \( X = \{X_1, X_2, \ldots, X_K\} \). Since coordinate information of anchor nodes is known, the distance among them is determined. The two-two distance among those anchor nodes form measuring matrix \( D = (d_{ij})_{K \times K} \); \( d_{ij} \) is the Euclidean distance between nodes. Similarly, in order to get product relationship between node coordinates, the coordinate inner product matrix \( B = (b_{ij})_{K \times K} \) is introduced. \( b_{ij} \) is shown in Equation (5).

\[
b_{ij} = (d_{ij}^2 + d_{ji}^2 - d_{ii}^2)/2
\]  

In the distributed positioning system, unknown nodes acquire parameter \( \alpha \), \( D \), and \( B \) in these two means:

1. Store all the three parameters in unknown nodes.
2. Send to unknown nodes via wireless signal.

In WSNs, for any unknown node \( K + 1 \), the ranging distance \( \bar{d}_{1,(k+1)}, \bar{d}_{2,(k+1)}, \ldots, \bar{d}_{K,(k+1)} \) of it between all other anchor nodes can be obtained by measuring technique. We assume unknown nodes can communicate with all anchor nodes. Because \( b_{ij} = b_{ji} \), \( B_{k+1} \) can coordinate the inner product matrix represented by the Equation (6):

\[
B_{k+1} = \begin{bmatrix}
    b_{k,(k+1)} \\
    \vdots \\
    b_{k,(k+1)} \\
    b_{k,(k+1)}, b_{k,(k+1)}, b_{k,(k+1)}
\end{bmatrix}
\]  

Like the centralized SV location algorithm, \( B_{k+1} \) is usually singular. There is at least one characteristic value of 0 when \( B_{k+1} \) is decomposed; this will result in the inverse matrix of the eigenvalue matrix not exists. Eigenvalue decomposition of
It is shown in Equation (7).

\[ B_{k+1} = (V_t, V_j) \begin{pmatrix} D_0 & 0 \\ 0 & D_2 \end{pmatrix} \begin{pmatrix} V_i^T \\ V_j^T \end{pmatrix} \] (7)

To sum up the derivation process, the distributed SV location algorithm includes the following steps:

Step 1: Translation of the reference node, so that a reference node is located in the origin of the coordinates.

Step 2: According to the actual coordinates of the anchor node, the distance between the anchor node is calculated, and the anchor node matrix \( D_k \), \( B_k \).

Step 3: The distance between the unknown node \( K+1 \) and all other anchor nodes is obtained by ranging technology \( d_{1(k+1)}, d_{2(k+1)}, ..., d_{k(k+1)} \). Using equation (6) generates the coordinates of the inner product matrix \( B_{k+1} \);

Step 4: To coordinate the inner product matrix \( B_{k+1} \) of feature value decomposition and take greater than zero in the characteristic value and the corresponding eigenvectors of the new coordinate inner product matrix.

Step 5: Equation (7) is used to calculate the coordinates of the unknown node \( K+1 \), and the translate to the original coordinate system.

3. Experimental Design and Discussion

In order to verify the performance of SV positioning algorithm, we need to test the proposed algorithm with RSSI based measuring technique, and then use Noisy Disk model to simulate the performance of SV positioning algorithm in many cases of distance measuring error.

3.1. Experimental platform and positioning result

The distance measurement technique based on RSSI can get the distance from the signal point to receive point with signal propagation attenuation model according to the strength or weakness of received RSSI signal. Since RSSI is susceptible to environment, there is no fixed propagation error model; instead, it needs to determine model parameters after an on-site survey as per actual environment.

We choose CC2430 as the main chip to make experimental nodes. CC2430 is the first 2.4GHz RF system single chip which complies with ZigBee technology. It still uses the infrastructure of CC2420 and integrates ZigBee RF front end, memory and micro-controller into a single chip. The current loss during its working is 27mA, which is respectively lower than 27mA or 25mA in receiving or transmitting pattern. Its’ hardware supports CSMA/CA function and digitized RSSI/LQI and strong DMA function. Each node can receive/send RF signal. When one node sends signal, the other nodes can receive it and surveys its RSSI. Here we estimate parameters of RSSI signal transmit error model in the laboratory.

3.1.1. RSSI signal transmit attenuation model [1]

Define \( P_y \) as the strength (mW) of signal which is sent by node \( j \) and received by node \( i \), and assume \( P_y = P_{yi} \); \( P_y \) is random variable of logarithmic normal distribution. \( P_y(dBm) = 10\log_{10} P_y \) complies with Gaussian distribution [4,9]. It is shown in Equation (8) and Equation (9).

\[ P_y(dBm) \sim N(P_y(dBm), \sigma_{yi}^2) \] (8)

\[ \bar{P}_y(dBm) = P_y(dBm) - 10n_y \log_{10}(d_y / d_0) \] (9)

Use CC2430 to make nodes to send and receive RF signal. Measure RSSI value to confirm the statistics model mentioned above. In order to get the above model, three parameters need to be determined, \( d_y \), \( P_y(dBm) \) and \( n_y \). Usually \( d_y = 1m \). \( P_y(dBm) \) is measured RSSI value when the distance \( d_y = 1m \) between node \( X_1 \) and \( X_2 \). When \( d_0 \) and \( P_y(dBm) \)
are both determined, put node $X_1$ and $X_2$ in different places. Then try several times to get RSSI value of them and substitute to Equation (9). After a few calculations, the mean value of $n_\beta$ is solved. Until this moment, three parameters of RSSI signal transmit attenuation model are all obtained. The red circle in Figure 1 is the average value of the RSSI signal measured at different distances from the sensor nodes. In this experiment, each position is measured 10 RSSI values, and then the average. According to the measured value, the parameters of RSSI signal transmission attenuation model can be $P_\beta(dBm) \approx -45$, $n_\beta \approx 3$. It is shown in Figure 1.

![Figure 1. Relationship between RSSI and distance](image)

The experiment finds that for two fixed sensor nodes far away at certain distance, not only can the RSSI fluctuate, but also with increasing distance. The vibration frequency and amplitude will become bigger. Figure 2 gives the distribution of signal strength along with time when two nodes of transceiver stand away at distance of 1, 2, and 3 meters. At each position, 105 groups of signals are observed; each group of signal at intervals of 2s. As seen, RSSI value undulates around the mean value at fixed position. The longer the distance, the greater the volatility. As shown in Figure 1 and Figure 2, the above mentioned model is in line with the actual situation.

![Figure 2. RSSI fluctuations with time at the distance of 1, 2, 3 m between two nodes](image)

3.1.2. Experimental result of SV positioning based on RSSI

After RSSI signal transmit attenuation model is achieved, it’s applied to evaluate the performance of distributed SV positioning algorithm based on RSSI distance measurement. The test scenario is chosen at laboratory. In the lab 10×10m², eight referential nodes are put around. There are devices and furniture indoor. An unknown node sends a signal that all referential nodes receive. Then, report the RSSI value of measured signal to a gateway in the form of wireless signal, which is forwarded by gateway to PC server for positioning calculation. Figure 3 shows the after 10 times calculated for average positioning results, which in the periphery of the square frame of eight red diamond for the reference node, within the 16 Blue solid rounds for the actual position of the unknown nodes, corresponding to 16 purple hollow circle based on the RSSI value is obtained by positioning the SV positioning results. The Euclidean distance between the actual position and the positioning position is used as the positioning error, the average location error of the 16 unknown nodes is 0.91m.

3.2. Simulation results

In the experiment, very few sensor nodes are employed. In the limited lab space range, the effectiveness of the proposed method is tested. However, one of the most significant features of WSNs is a big node scale, wide deployment space. In practical use, hundreds of thousands of sensor nodes are deployed. In environmental condition, we have no way to deploy such large-scale WSNs to test the performance of positioning algorithm. Here, we take software simulation mode to assess quantitatively the effectiveness of the positioning algorithm. This not only sets a lot of sensor nodes and meeting requirements of wide application, but also the simulation can be implemented that factors affecting the positioning of WSNs exist in reality such as measuring distance error, node communication distance.
Choose Matlab as the simulation platform to analyze and evaluate the performance of SV positioning algorithm. All methods are compiled with Matlab language, which is appraised at four aspects such as number of anchor node, communication range of sensor nodes, number of nodes and measuring distance error. For the first three elements, they can be adjusted through increasing node number and enhancing sensor node’s communication range. With regards to measuring distance error, since there is a few distance ranging techniques and the measuring distance errors of different techniques differ largely, the measuring distance between nodes can be obtained through Noisy Disk model. It is shown in (10).

$$\tilde{d}(x_i, x_j) = \begin{cases} N(d(x_i, x_j), \sigma) \\ \text{Inf} \end{cases}$$ (10)

SV positioning algorithm can realize centralized and distributed positioning. At first, we conduct simulation evaluation of centralized SV positioning algorithm. About the arrangement of WSNs, there are regular deployment and random deployment. To assess the impact of anchor number on centralized positioning algorithm, Figure 4-5 present positioning results of the two deployment methods. The red asterisk is the position of anchor node, hollow circle is the actual coordinate of unknown nodes, and the straight line connected with circle refers to the distance between unknown node’s actual position and estimated position. Two experiments suppose all nodes can mutually communicate; the measuring distance error is 15%.

In Figure 4, 81 unknown nodes are deployed in the area 100×100m²; the number of anchor is respectively 4, 7, 10 and 13. Locate all unknown nodes and get the positioning error: 3.43m, 3.71m, 3.96m and 3.42m, respectively. In Figure 5, randomly deploy 80 nodes in the area 100×100m²; choose 4, 6, 8 and 10 nodes as anchor nodes. These nodes are used as unknown nodes to do positioning test. The positioning error is obtained separately: 3.40m, 2.96m, 3.12m and 3.39m. From two groups of experiments, we realize whether rule deployment or random deployment, the number of anchor nodes hardly impacts positioning error.

Figure 3. Experimental localization result

Figure 4. Localization error results of regular deployment. The ranging error is 15%, and the anchor node numbers are 4, 7, 10, 13, respectively
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We conduct a second group of simulation tests to find out the relationship between network connectivity and centralized SV positioning result. In the first group of experiments, suppose all nodes can communicate mutually. It should be noted that in practical use, it’s not possible for all nodes to communicate mutually, especially when deploy area is big and/or node communication distance is small. When direct communication between nodes can’t be achieved, the shortest path between them is utilized to calculate the measuring distance between them. In order to solve the shortest path between two nodes, we use graph G to represent the entire network; sensor node in the network is vertex of graph G. Suppose the weighted matrix of graph G is H. When there is one edge between vertex \( X_i \) and \( X_j \), i.e. \( X_i \) and \( X_j \) are in communication range, \( H_{ij} = 1 \); otherwise, \( H_{ij} = 0 \). That’s because it’s assumed all nodes communicate at the same distance. If node \( X_i \) is not within \( X_j \) ’s communication range, node \( X_j \) is not in \( X_i \) ’s communication range too. Therefore, weighted matrix H is a symmetric matrix; we can get in Equation (11).

\[
H_{ij} = \begin{cases} 
1, & \text{if } d(x_i, x_j) \leq CD \\
0, & \text{if } d(x_i, x_j) > CD
\end{cases}
\]  

(11)

With weighted matrix and node measuring distance within communication range, we can define the shortest path of all nodes like in Equation (12).

\[
\bar{d}_{\text{min}}(x_i, x_j) = \min \{\bar{d}(x_i, x_j) \times H_{ij}, (\bar{d}(x_i, x_k) \times H_{kj} + \bar{d}(x_k, x_j) \times H_{kj})\}
\]  

(12)

To start, it tested the node’s network connectivity and relative positioning error results in different communication ranges. In the area 100x100m², 70 unknown nodes and 10 anchor nodes are arranged. Suppose the error of surveyed distance among all sensor nodes is 15% and all nodes’ communication range is consistent. By changing node’s communication distance, network connectivity can be adjusted. Node communication distance CD is made 30m, 40m, 50m and 60m; network connectivity is 17.525, 25.675, 38.750, 47.500, and relevant positioning error is 9.02m, 5.72m, 4.39m and 2.93m. Figure 6 gives network connectivity and the positioning result under the above premises. With CD growing up, network connecting lines increase and node’s positioning error bars become shorter.

In order to better evaluate the performance of the centralized SV location algorithm, the following were from the deployment of sensor nodes number (NN), the ranging error between nodes (\( \sigma \)), node communication distance (CD) positioning precision angle to view the algorithm, as an evaluation criterion for the performance of the proposed localization algorithm in the paper. First, the effect of NN on the performance of SV location algorithm is evaluated. Figure 7 (a) is in the case of \( \sigma = 0.01 \), to add the sensor nodes to the network, the positioning error of the SV algorithm is obtained. It can be seen that when the NN is more than 40, the position error of the three CD is decreased. When NN increases from 70 to 100,
the three curves become more and more smooth. In the case of NN=70, CD=50, the average positioning error is 3.85m, NN=100, the mean position error is 3.23 m, and 0.62 m is reduced. Figure 7 (b) gives the sigma changes, the relationship between average positioning error and NN, similar to Figure 7 (a), the slope of the curve between NN 30 to 40 is greater than the slope of NN between 40 and 70, and the slope of the curve between NN 40 and 70 is larger than that between 70 and 100. When is the NN>70, the location error is small, which provides a reference for the deployment of the sensor network. From Figure 7 (b) can also be seen that influence of \( \sigma \) on SV positioning is relatively large (spacing between the three curves obviously). In the case of NN=80, \( \sigma =0.1, 0.2 \) and 0.3 average positioning error values were 3.26m, 6.11m, 8.52m; the positioning accuracy decreased significantly.

![Network Connectivity Diagrams](image1)

![Location Error Diagrams](image2)

Figure 6. Network connectivity (left) and location error diagrams (right) at different CD

![Average Location Error Diagrams](image3)

Figure 7. Average location error as a function of the sensor node number at different communication distances (a) and different ranging error (b)
Secondly, compare the effect of measurement error on the performance of a node localization algorithm. It is shown in Figure 8(a). The variation of NN for 70 and 100 curves was slower than that of NN for 40. When $\sigma$ rises from 40% to 50%, the positioning error was increased by 7.50m, 6.63m, and 10.92m. It also verifies the relationship between the number of nodes and the location error in the upper group experiment.

4. Conclusions

This paper proposed a new localization algorithm based on a set of statistical vectors. The algorithm is solved by simplifying the center coordinate matrix by the translation node coordinate system to get the coordinates of the inner product matrix directly. By using statistical vectors sets to construct the inner product matrix of coordinates to realize the noise measurement of ranging noise, it can effectively reduce the positioning error caused by the ranging error. In this paper, the positioning algorithm based on statistical vector set of centralized localization, distributed localization scheme, and distributed localization algorithm is applied to the positioning experiment based on RSSI ranging technique. It is proved that the algorithm is effective.

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